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History of Physics (22)

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Joseph Fourier and Sadi Carnot: a counterpoint in 19th century study of heat

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I take the opportunity of this year celebration of the two hundred and fiftieth anniversary of Joseph Fourier to discuss his major achievement, the treatise *Analytic Theory of Heat* (published in 1822), in the context of early nineteenth century conceptions of heat. As an interesting comparison, I shall pay also attention to an almost contemporary work of another hero of modern theory of heat, Sadi Carnot's *Reflexions on the motive power of heat*, which appeared in 1824. As the title of my paper makes it explicit, I intent to present both men and their science using the metaphor of a counterpoint which suggests that I shall emphasize the differences. However, according to definitions, a counterpoint is "the combination of two or more independent melodies into a single harmonic texture in which each retains its linear character". Thus, I shall also put forth how both men realized genuine breakthroughs because their approaches embodied the same attitude with respect to the nature of heat. But first things first, let us introduce our main characters.

Joseph Fourier, born in 1768 in the burgundy town of Auxerre and died in 1830 in Paris, is a perfect case of a supremely gifted young man of humble origin whose achievements became a highlight of 19th century science and who reached the top of the scientific establishment. Son of a tailor, he lost both his parents at an early age and his ensuing education was taken care of by the clergy. After establishing local reputation, he moved in 1794 to Paris where he was taught by such scientific authorities as Gaspard Monge, Joseph Louis Lagrange or Pierre Simon de Laplace. Because of his rising scientific reputation he was proposed in 1898 to take part in the famed Egypt expedition led by the future emperor Bonaparte. He proved there that beside his scientific skills, he was also an able organizer and administrator. After his return to France following the French defeat in 1801 he planned to come back to scholarly activities but Napoleon decided otherwise when he "asked" Fourier to become the prefect of the Isère department. This was a heavy burden but Fourier did not disappoint Napoleon confirming his administrative skills. More surprisingly, he did not give up his research interests either. Quite to the contrary, as it is in this period that he did most of his investigations paving the way to his *Analytical Theory of Heat*. After Napoleon's defeat and the restoration of the monarchy Fourier came back to Paris but initially did not fare well because of his Bonapartist past; he went however eventually elected to the French Academy and became in 1822 its secretary, a position he held until his death.

While initially criticized (more on this soon), *The Analytical Theory of Heat* became eventually widely acclaimed and a model to follow in any positivistic-mathematical approach to the investigation of nature. Also, because of its remarkable use of revolutionary mathematics, Fourier's treatise contributed much to define the field of mathematical physics.

Sadi Carnot was born in 1796 in Paris where he died in 1832. He was, in contrast with Fourier's modest origins, the

son of one of the most famous men of his time, the general Lazare Carnot, known both as a gifted scientist and a brilliant revolutionary leader. Sadi enlisted at 16, the minimum age possible, in the *Ecole Polytechnique* where he obtained a first rate scientific and military training. After two more years spent at the *Ecole du Génie* at Metz, he served as military engineer in the French army. Frustrated by his situation, he managed to join, in 1819, the General Staff Corps in Paris but soon asked for a leave. Except for a short return to military duties followed by definitive retirement (1828), Carnot spent the rest of his short life studying while living from a rather modest rent. In 1821, on the occasion of a stay with his father and brother exiled in Germany, Carnot turned his attention to steam engines. The result of his pondering on heat was published in 1824 and went almost unnoticed. It received due attention only after Carnot's death, when Emile Clapeyron, William Thomson and especially Rudolf Clausius saved it from oblivion and explained to the community Carnot's merits. Carnot's treatise contains, as we shall see, indeed much of the founding material for the second principle of Thermodynamics.

To properly appreciate Fourier's and Carnot's contributions to the theory of heat, one needs to recall the context and the situation, at the time, of the French science and its institutions. Flourishing all over the 18th century, French science reached its apogee at the turn of the 19th century. Benefiting of Napoleon's keen interest in science, it was endowed in the last decade of the century with prestigious institutions such as the *Ecole Normale Supérieure* (created in 1794) and the *Ecole Polytechnique* (1795). Together with the *Académie Royale des Sciences*, reorganized and renamed the 1st class (*classe des sciences physiques et mathématiques*) of the *Institut National* during the French Revolution, these institutions had the lead in French scientific education and research. The positions offered there represented consequently sure steps to the pinnacle of scientific careers. Because of the tightly centralized organization of the French empire, control over these institutions, the nominations boards, the themes of prizes, enabled, as a matter of fact, almost full control over scientific beliefs and paradigms. In the time period of interest to us, the dominant science was what historians have since aptly named *Laplacian science*. Its most distinguished dignitaries were men many of them still remembered for first rate contributions: Pierre Simon de Laplace of course, but also Claude-Louis Berthollet, the French chief chemist, and their brilliant "protégés", Jean Baptiste Biot, Siméon Denis Poisson, Etienne Malus, François Arago, to name only a few.

The dominant paradigm: Laplacian science

To understand what Laplacian science was, one must first recall the success of Newton's mechanics and of his theory of universal attraction (exposed in Newton's *Mathematical Principles of natural Philosophy*, 1787) which greatly impressed the natural philosophers of the time. When New-

ton's physics eventually overcame on the continent the rival system of Descartes, many thought that the keys to an understanding of all of natural phenomena, not only mechanical but optical, electrical and magnetic, etc. were at last in sight. Indeed, Newton's theory of attraction but also his (rather wild) speculations exposed in the rhetoric form of questions (Queries) at the end of his treatise *Opticks* first published in 1704 suggested that all of phenomena resulted from the mutual actions of minute particles composing matter and possibly some (imponderable) subtle fluids. This inherently corpuscular, mechanistic worldview, with dynamics derived from inter-corpuscular forces, truly blossomed at the end of the 18th century. It was much helped by the development of analysis and its expert handling by some of the most brilliant minds of the time.

Among the latter, one finds Pierre Simon de Laplace, most gifted mathematician, astronomer and physicist, the author of many treatises proposing sweeping perspectives over natural phenomena at the astronomic, terrestrial, macroscopic or microscopic scale. His friend's Berthollet's summer residence in Arcueil (near Paris) gave the name to an informal circle of scholars, the *cercle d'Arcueil*, sharing Laplace's views over the proper conduct of natural inquiry and fully dedicated to the cause. Laplacian science is characterized first by its basic reliance on Newtonian mechanics and the explanatory scheme offered by Newton's theory of universal attraction. More precisely, the Laplacians believed (to various degrees), within a corpuscular conception of matter, that its constituents repel or attract themselves because of, besides gravity, of other (possibly short range) forces. Moreover, some "fluids" composed of minute corpuscles were postulated which, interacting, always with central forces, were held responsible for electrical, magnetic, optic, etc. phenomena. In Laplace's best supporter Jean-Baptiste Biot's words:

In order to explain [the respective phenomena of electricity, magnetism, and heat] physicists have imagined certain elastic fluids endowed with attractive or repulsive properties... that are named electric fluid, magnetic fluid, and the principle of heat, or caloric. By means of these suppositions most of the phenomena are able to be represented up to a certain point.

(from his article « Sur l'esprit du système » in *Melanges scientifiques et littéraires*, Paris, 1858)

The "Laplacian" touch to this neo-newtonian world-view consisted in the sheer mathematical virtuosity in which excelled Laplace and many of his supporters such as Poisson or Biot: to reach an explanation of macroscopic effects, the Laplacians summed over the minute microscopic actions using integration with maestria.

This paradigm was quite powerful to tackle a surprising range of phenomena (so that one can consider it as an example of what Thomas Kuhn defined as "normal science"). Not only did it lead to spectacular results in celestial mechanics (which is no surprise given that this was the defining field of the paradigm): Laplacian physics could bring as well some insights on such diverse topics as capillarity or optics. Among many phenomena investigated, there was also heat.

Early heat phenomenology: contributors and milestone

Let us then review briefly what was known about heat at the turn of the 19th century. The first thermometers appeared at the very end of the 16th century and it took a long time before an understanding of what temperature is and how it relates to heat content. Two key observations/assumptions should be quoted here, the so-called "Newton's law of cooling" (1701) which loosely states that the rate of temperature change is proportional to the difference of temperature of the relevant bodies, and the even more basic statement of the Dutch Herman Boerhaave that temperatures tend spontaneously to equalize, called the law of the "equilibrium of heat" (1720). In the 1760's the Scot Joseph Black contributed further his very important observations on the different capacities of bodies to stock or release heat which brought him to the definition of specific heat. He also drew attention to the phenomenon of latent heat: because there was no temperature change observed during evaporation while heat was added to the body, the later was described as "concealed" or "latent" (an understandable characterization in the framework of a substantialist theory of heat, to be discussed next). Specific heats were later investigated by Laplace and Lavoisier in the framework of their calorimetric experiences (1789).

The "substantialist" conception of heat

In the framework of Laplacian science taken at face value, heat was thought of as the manifestation of an imponderable fluid aptly named by Lavoisier as "Caloric". As already hinted at, the (minute) particles of this fluid repel from each other but are instead attracted by those of matter and tend to agglutinate around the latter. This is what heating bodies is all about. As a result, molecules of matter are spread apart which can account in a simple way for thermal dilation. In this substantialist conception, thermometers are to be understood as genuine gauges measuring the level of the caloric within bodies. This does not go without paradoxes. Besides the phenomenon of latent heat, it was also observed that bodies do not gain weight while heated so that one had to assume that caloric was weightless hence "imponderable".

The "dynamical" conception of heat

There were other paradoxical aspects of the caloric: The simple phenomenon of friction appeared quite puzzling too: the amount of heat one could "extract" from a rubbed body seemed unlimited so that either the latter had an infinite content of heat (then necessarily caused by a brusque variation of its specific heat) or heat could be created at will "out of blue", which was judged a strange conclusion indeed. Benjamin Thompson (better known as count Rumford) concluded an account of his famous observations of the prodigious amount of heat friction could generate (the boring of bronze artillery guns) with the following words:

in reasoning on this subject, we must not forget to consider that most remarkable circumstance, that the source of the heat generated by friction, in these experiments, appeared evidently to be inexhaustible. It is hardly necessary to add, that anything which any insulated body, or system of bodies, can continue to furnish without limitation, cannot possibly be a material substance: and it appears to me to be extremely difficult, if not quite impossi-

ble, to form any distinct idea of anything, capable of being excited, and communicated, in the manner the heat was excited and communicated in these experiments, except it be Motion.

(An Inquiry concerning the Source of the Heat which is excited by Friction, *Phil. Trans. R. Soc. Lond.*, vol. 88, 80-102, 1798)

Rumford did not venture to speculate on the kind of motion that was involved and how the phenomena of heat were related to it. He was not the first to relate heat to motion and to promote thus a “dynamical” conception of heat but it still took some decades before the substantialist theory gave way to the dynamical one and even more to see heat identified with the energy of thermal motion in the early kinetic models of gases of Joule, Clausius, Kelvin, and mature investigations of Maxwell and Boltzmann. Clearly, this shift in favor of the dynamical theory occurred in tight relation with the recognition of the principle of conservation of energy discovered around mid-century.

Between commitment and agnosticism as to the nature of heat

I hinted at the dominant character of the substantialist, caloric, theory of heat at the turn of the 19th century. However, it would be wrong to assume that those who put it to work in their speculations were unconditionally committed to its reality. One finds at the end of the 18th century a fair amount of statements where the caloric is presented merely as a handy model to think of heat phenomena while an agnostic attitude is expressed in what concerns heat’s true nature. To witness, the following lines from Lavoisier’s celebrated *Traité élémentaire de chimie*, 1789, who yet identified the caloric among the 33 elements of his new chemistry:

Wherefore, we have distinguished the cause of heat, or that exquisitely elastic fluid which produces it, by the term caloric. Be sides, that this expression fulfills our object in the system which we have adopted, it possesses this further advantage, that it accords with every species of opinion, since, strictly speaking, we are not obliged to suppose this to be a real substance; it being sufficient, as will more clearly appear in the sequel of this work that it be considered as the repulsive cause, whatever that may be, which separates the particles of matter from each other; so that we are still at liberty to investigate its effects in an abstract and mathematical manner.

Or, this time quoting from R. Haüy’s *Traité élémentaire de physique* (1803):

Without any thing to decide between [the caloric and the dynamic theories], we adopt the language which is consistent with the [caloric fluid one], while regarding it solely as an hypothesis more suitable to assist the understanding ()

And, from the already quoted article by Biot:

Also true physicists acknowledge the consideration of fluids [the electric, magnetic, caloric] uniquely as a convenient hypothesis, to which they rightly refrain from attaching ideas of reality, and that they are given to modifying or entirely abandoning when facts will prove them to be inconsistent

A crucial break in the study of heat: giving up ontological questions

Even if there was thus a fair amount of agnosticism when true nature of heat was considered, in practical studies of heat effects, and especially in attempts at quantitative results, it was the caloric model that was used. I am now in position to explicit the counterpoint established by Fourier’s *Théorie analytique...* and Carnot’s *Réflexions...* But first, what is the common “harmonic structure” this counterpoint is built upon ? Both works share the same attitude with respect to the ontological question of the nature of heat: they just do not care and introduce instead new deductive schemes, independent of what heat truly is, as they rely only on heat effects. Indeed, Fourier is interested in the laws and mathematical expressions of heat exchanges between bodies causing changes in temperature, and Carnot in heat engines producing motive power. But, just because of this, their works build a counterpoint: in modern terms, Fourier considers in effect pure dissipation while Carnot is interested in the production of motive power which, as he will recognize, necessitates, to be optimal, to avoid spontaneous passages of heat between concomitant parts of the engine. So, here we have it: dissipation without work on one hand, and work without dissipation on the other, a beautiful counterpoint indeed.

Fourier’s motivations

What were Fourier’s motivations underlying his interest in heat? A folk account suggests that because of a disease contracted in Egypt that made him extremely sensitive to cold, problems related to temperature changes came naturally under his scrutiny. There is a more documented and compelling explanation. It has to do with Fourier’s ambition to challenge Laplace’s universal mechanistic worldview. Indeed, while heat is as universal as gravitation, its phenomenology is not addressed in Laplace’s monumental *Système du monde*. Obtaining a theory of heat exchanges and ensuing temperature changes meant consequently to Fourier to equal in scope and universality the physical vista of his main scientific opponent Laplace. The ambition of Fourier is clear since page one of his treatise:

PRIMARY causes are unknown to us; but are subject to simple and constant laws, which may be discovered by observation, the study of them being the object of natural philosophy.

Heat, like gravity, penetrates every substance of the universe, its rays occupy all parts of space. The object of our work is to set forth the mathematical laws which this element obeys. The theory of heat will hereafter form one of the most important branches of general physics.

(*Théorie analytique*, Discours préliminaire, p. 1)

Fourier emphasizes further in the long *Discours préliminaire* how heat escapes the laws of mechanics and is hence in need of new physics and mathematics. To witness the following quotes:

[Archimedes] explained the mathematical principles of the equilibrium of solids and fluids. About eighteen centuries elapsed before Galileo, the originator of dynamical theories, discovered the laws of motion of heavy bodies. Within this new science Newton comprised the whole system of the universe. The successors of these philos-

ophers have extended these theories, and given them an admirable perfection [...] But, whatever might be the scope of mechanical theories, they do not apply to the effects of heat. The latter encompass a special order of phenomena which cannot be explained with the help of the laws of motion and equilibrium

And further:

[...] a very extensive class of phenomena exists, not produced by mechanical forces, but resulting simply from the presence and accumulation of heat. This part of natural philosophy cannot be connected with dynamical theories, it has principles peculiar to itself, and is founded on a method similar to that of other exact sciences. The solar heat, for example, which penetrates the interior of the globe, distributes itself therein according to a regular law which does not depend on the laws of motion, and cannot be determined by the principles of mechanics. The dilatations which the repulsive force of heat produces, observation of which serves to measure temperatures, are in truth dynamical effects; but it is not these dilatations which we calculate, when we investigate the laws of the propagation of heat.

(Chapter I, section 17, p. 23)

Finally, Fourier makes plain how only mathematical analysis of the kind foreign to mechanics, can make the investigation of the *Théorie analytique* reach its aim:

The effects of heat are subject to constant laws which cannot be discovered without the aid of mathematical analysis. The object of the theory which we are about to explain is to demonstrate these laws ; it reduces all physical researches on the propagation of heat, to problems of the integral calculus whose elements are given by experiment. No subject has more extensive relations with the progress of industry and the natural sciences ; for the action of heat is always present, it penetrates all bodies and spaces, it influences the processes of the arts, and occurs in all the phenomena of the universe

(Chapter I, Introduction, p. 14)

Carnot's motivations

It is tempting, given the lack of recognition his work suffered, to fancy Carnot's *Réflexions* as an outcome of a solitary quest, as a work of in isolated visionary much in advance of his time. But this would be neglecting Sadi's father Lazare's most important work in theoretical engineering. In 1783, in a context where applied science and engineering gains notably in importance Lazare Carnot publishes a remarkable essay on machines, his *Essai sur les machines en general*. This is a close examination of the principles that govern the efficient transmission of motive power in mechanical engines. Lazare recognizes in particular the fatal effects of impacts between connecting rods, teeth of cogwheels, etc., when the later are not "matched motionwise". Sadi's declared intent, namely to found a general theory of heat machines, bypassing in scope and generality the purely empirical results obtained from the practice of steam engines appears a logical continuation of his father's interest in founding a general theory of mechanical engines. Also, the impairments (leading to losses of live force) that the father recognized as the main obstacle to the efficiency of

mechanical engines is in perfect analogy to dissipation (irreversibility) that prevents maximal efficiency of heat engines according to the son's *Réflexions*.

A fundamental input to Fourier's theory: Prévost "theory of exchanges"

Fourier bases his theory on what is dubbed at the time the "theory of exchanges" due to the Geneva professor Pierre Prévost and exposed in his "Memoire sur l'équilibre du feu" (*Journal de Physique*, 38 ,1791). Fourier clearly pays his dues to Prévost declaring:

[...] bodies mutually transmit heat [...] It is in this exchange of [heat] rays that the hypothesis proposed by Professor Prevost of Geneva principally consists. This hypothesis furnishes clear explanations of all the known phenomena. It lends itself more easily than any other [hypothesis] to the applications of calculations: it therefore appears useful to us to choose it, and it can even be employed with profit to represent the manner of propagation of heat within solid bodies.

(Fourier, from a paper he submitted as an entry to the 1811 Institut's prize devoted to the study of heat propagation.)

Prévost proposed his hypothesis as a solution to the riddle of the apparent transmission of "cold" that could be observed in a "reverse" of the experiment of "radiant" heat reflected and focused on a distant thermometer with the help of parabolic mirrors. By the end of the century, another famed Geneva scholar, Marc-August Pictet replaced the incandescent coal bricks in the focal point of the mirror by a recipient full of ice. The distant thermometer recorded a temperature decrease. Some of the contemporaries did not hesitate to fancy that this experiment proved the existence of rays of cold! Prévost explained instead that because all bodies radiated heat whatever their temperature, the correct explanation of the thermometer fall was that the latter was sending more than it was receiving.

Another ingredient of Fourier's theory of heat is Newton's law of cooling, or rather Fourier's analytical expression of the latter that states the proportionality of heat flux between contiguous portions of matter to the gradient of temperature. This was enough for Fourier and his mathematical expertise to immediately obtain the celebrated equation of heat propagation.

Most of the rest of his treatise is devoted to apply the theory to various cases with specific boundary conditions, and of course to the discussion of the general technique to obtain solutions that he derived using trigonometric series. Because my primary concern is the physical theory of heat, I shall leave aside this remarkable mathematical discovery and all that it prompted in 19th century mathematics to turn now to Carnot's *Réflexions*.

A fundamental input to Carnot's theory: the watermill as a model for the heat production of motive power

Taking the *Réflexions* at face value, Carnot assumes the substantialist theory of heat. However, a closer scrutiny shows that it is not really material to his reasoning, at least as far as one focuses on the essential results obtained in his memoire. Indeed, the originality of Carnot's deductions

resides in the fact that they dispense with any hypothesis about heat's very nature. First, to obtain motive power using heat, Carnot emphasizes that one needs a difference of temperature. Then, it is the transfer of heat from hot (furnace) to cold (condenser) that generates motive power: according to Carnot, heat "falling in temperature" generates work in perfect similarity to (gravitationally) falling water. Hence, the production of motive power in a heat engine is obtained as if a variation of a kind of "thermic potential" was compensated by a gain of mechanic energy. This is of course wrong from the point of view of the principles of thermodynamics. However, the elegant reasoning of Carnot on "his" ideal cycles, linking efficiency to irreversibility, and his recognition of the fundamental need, in the production of motive power, of a heat transfer across a temperature difference (again, insights independent of any hypothesis on the nature of heat) will prove, once properly understood, crucial for future progress.

A second counterpoint: the contrasted receptions of Fourier's and Carnot's works and their later fate

In 1807, Fourier sent a paper devoted to heat propagation to the First class of the Institute. In spite (or rather because) of the revolutionary character of its content, the paper was (to make it short) rejected, mainly due to the hostility of the Laplacians and Lagrange's frustration with the lack of proper justification for Fourier's innovative mathematics. Fourier did not give up and over the consecutive years kept trying to make his theory respectable. In particular in 1811 he won the Institute's prize for heat propagation but his theory remained unpublished until 1822. Benefiting from a weakening of the Laplacian grip over French science, Fourier's physics went finally hailed as a model for a successful account of natural phenomena. What the Laplacians considered its major flaw, its dispensing with any mechanistic model, went recognized as its essential virtue. It avoided any unnecessary speculation on the very causes of heat and was a potential manifesto for a positivistic investigation of nature. Fourier's mathematical techniques and the assumptions behind nourished, on the other hand, a great deal of 19th century progress in mathematics. Fourier enjoyed lastly a deserved fame and respect which is certainly witnessed by his election as secretary of the restored Royal Academy once his role in Napoleon's empire went forgiven.

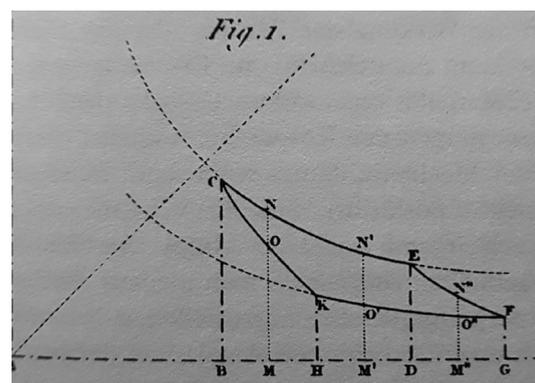
It took Fourier's work several years to prevail and set the standards for a new approach to phenomena, but it is nothing in comparison to the neglect which hampered Carnot's *Réflexions* until they were rediscovered by men able to understand their scope and value. At the time of its publication the *Réflexions* went ignored, presumably because it appeared too theoretical and abstract to some, while too odd and remote from contemporary physics to others. Indeed, practitioners of steam engines looking for ready-made solutions to improve their machines failed to find any in the general theory of Carnot. The scientific establishment, on the other hand, if it read Carnot at all, must have been rebutted by Carnot's highly original style of reasoning totally at odds with the contemporary ways of mechanistic-analytical schemes of proof. There is only one known exception to this neglect: in 1834 the engineer Emile Clapeyron reformulated Carnot's theory in mathematical terms (we owe him for instance the

expression of Carnot's cycles in terms of pressure-volume diagrams), but did not contribute any insight of his own and, most of all, failed to see Carnot's theory true potential and meaning. Carnot did not live up to see his theory finally arise the interest it deserved: by the time Clapeyron reformulated his work, he was already dead. The seeds of future progress were however planted and ready to grow. The young William Thomson learned of the work of Carnot while reading Clapeyron during his Parisian sojourn in 1847. In turn, Rudolf Clausius learned of Carnot reading Thomson. Both, and especially Clausius, took the work seriously enough to bring it back to the forefront of science. In 1850 Clausius showed how to make Carnot's conceptions easily compatible with the just accepted principle of the conservation of energy. In a rather generous move, Clausius further granted to Carnot most of the merits in the discovery of the facts leading to the second principle of thermodynamics. Indeed, he emphasized how crucial was Carnot's observation that there is no conversion of heat into work without a portion of heat transferred across a difference of temperature.

A posteriori, Carnot went eventually vindicated. But this is not all: at the end of his life, Carnot actually corrected his initial views on the production of work without consumption of heat! His brother Hippolyte who assisted Sadi in the publication of the *Réflexions* and who later brought order to the unpublished notes of his defunct brother transmitted some to the Academy. These documents show beyond doubt that Carnot had understood, in the years consecutive to the writing of his *Réflexions*, that not all the heat taken from the hot source was given back to the cold one if work was obtained. Thus, to many historians of the first principle (most notably Thomas Kuhn) Carnot appears an unsung hero of the conservation of energy and thus, even more a founder of Thermodynamics.

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Carnot circle-process for a gas, from a treatise of Emile Clapeyron (*Journal de l'Ecole Polytechnique* XXIII: 190, 1834).