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Milestones in Physics (14)

The centennial of Richard P. Feynman

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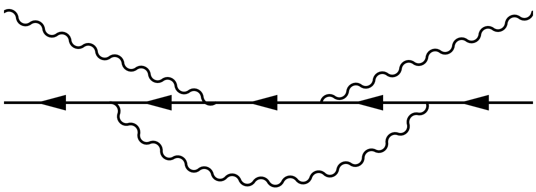
This year the physics community celebrates the centennial of the birth of Richard Feynman. Once one reads a bit about the man, as I had to do for the preparation of this note, it becomes less clear that he would find that much ado appropriate, as he claimed that "ancestor worship" hinders a generation from appreciating its own time. In any event, the writer and the reader have no choice other than worshipping him a bit together. Let us do so by going right to the heart of the matter.



Feynman's name became inseparably linked to Feynman diagrams, like the one seen in the figure. That particular diagram represents a contribution to the amplitude of a photon scattering off an electron (Compton scattering), an experiment simple enough to be shown in class (it was e.g. shown to me by the late Valentine Telegdi). Other, similar diagrams account for the determination of the magnetic moment of the electron, i.e. of its gyromagnetic factor g, in terms of the fine structure constant α. The best experimental value

$$g/2 = 1.00115965218073(28),$$

allows for a determination of α that is ten times better than, but in agreement with, the next best method. It makes Quantum Electrodynamics (QED) the physical theory that is verified to the greatest degree of precision (largest number of significant digits). The expression of "seeing further by standing on the shoulders of giants" can hardly be used more appropriately than in relation with those little drawings. They enabled generations of theoretical physicists to compute testable predictions of QED and of many other field theories, in particle and in condensed matter physics, in an almost automated way.

Such diagrams have two rather different faces. One the one hand they are a means to compute correlation functions of a *quantum field*, in that they organize the perturbative expansion in some coupling parameter, such as α, by taming the combinatorics, and allow to obtain finite results by renormalization. On the other hand, and much more pictorially, they represent *particles*, real and virtual ones, in guise of colliding trajectories with particles and antiparticles being drawn alike, except for running in opposite time directions; this is the case in the figure for electrons and positrons, shown as straight lines, but not photons (wiggly lines), as they are their own antiparticles. That dual reading is common knowledge nowadays (after all particles are asymptotic states of the quantum field), and is largely due to Feynman. His work on QED was rewarded with the Nobel prize in Physics in 1965, which he shared with Sin-Itiro Tomonaga and Julian Schwinger, whose approaches were quite different.

In order to understand how the diagrams came into being, let us backtrack a bit. Feynman was born on May 11, 1918 into a family that was not so well-off. He grew up in Far Rockaway (Queens, New York) and was raised by parents of Belarus and Polish ancestry. Richard's inquisitive attitude was fostered early on by his father, who wanted him to become a scientist. At high school he learnt to become a "real guy" and not a "sissy"; in his own words: "To be a practical man was, to me, always somehow a positive virtue, and to be 'cultured' or 'intellectual' was not". (He later admitted that "The first was right, of course, but the second was crazy.") That included his direct manners and his disrespect for authorities of all kinds. He also taught himself some advanced mathematics from a book by the title *Calculus for the Practical Man*. After high school he went to MIT in 1935 to study mathematics as his major subject. Not before long he went to see the head of the math department to just ask him "What is the use of higher mathematics besides teaching more mathematics?" Upon being told "If you have to ask that, then you don't belong to mathematics", he simply followed the advice and switched to engineering, which he soon found to be too practical to his taste. So he ended up doing physics, which was the real thing. All this, by the way, did not prevent him from taking part in 1939 in the Putnam mathematics competition, open to all undergraduates in the U.S. and Canada, where he ranked among the five (not further ranked) top scorers. But it shows, once again, that he loved math as long as it was problem oriented and led to concrete results.

By 1939, when Feynman arrived in Princeton for graduate studies, he already had set himself the goal of solving the problems with QED. Eventually he got his adviser Wheeler, by a few years his senior, interested in the idea for some time. Before continuing however we have to backtrack once more and briefly review the status of quantum field theory back then. Attempts to understand the theory were under way already for a while. In a sense, it was nothing but obvious from locality (the German word *Nahwirkungsprinzip* is more precise) that quantum theory would have to apply not just to particles, but also to fields, like the electromagnetic one. As early as 1929 Pauli and Heisenberg realized that the theory was plagued with divergences, the origin of which had to be found in the infinitely many degrees of freedom interacting with a point-like particle, not unlike with the self-interaction of a point particle in the classical theory. This did not prevent some significant successes, though all just obtained in leading order of perturbation theory. For instance Weisskopf and Wigner (1930) calculated spontaneous emission and thus the natural line width of spectral lines, as well as Einstein's A,B-coefficients; Bethe and Fermi (1932) understood that the interaction between charged particles is mediated by the exchange of virtual photons (later to be extended to nuclear forces by Yukawa); Bethe and Heitler (1934) treated the electron and the positron as a Dirac particle and were able to calculate effects such as bremsstrahlung of relativistic particles.

Feynman's initial approach to QED was both heretical and radical. First of all he decided that the infinities had to be removed at the classical level first; second, he just denied the existence of the electromagnetic field as a carrier of degrees of freedom, demoting it to nothing more than an auxiliary quantity determined by the particle variables, as we are used to in *electrostatics*. There, the interaction of a particle with itself can be declared to vanish and the mutual ones attributed to an action at a distance (*Fernwirkung*); and so did Feynman for *electrodynamics*. In order to comply with relativity, he would simply replace the action at a distance in *space* by one in *spacetime*. He soon realized that there would be no radiation resistance, but Wheeler (using an observation of Dirac) noticed that it could be rescued by treating emission and absorption of radiation (time-)symmetrically (Wheeler-Feynman electrodynamics). That however made the theory even more heretical because, if one thinks it through, one should no longer talk of emission and absorption, but just of transmission of radiation; in other words any light, that was once emitted by some star and is presently on a journey, is somehow predestined to be absorbed some day. (Crazy as it sounds for real photons, this is however the fate of virtual ones.) Quite importantly the theory also allowed for an action principle: The action is a functional of the worldlines $x^{(i)}(\cdot)$ of the particles i of masses $m^{(i)}$ and charges $e^{(i)}$ (no field variables!) given by

$$S = \sum_i \int m^{(i)} ds^{(i)} + \sum_{i \neq j} \iint e^{(i)} e^{(j)} \delta(R_{ij}^2)\, dx_\mu^{(i)} dx^{\mu(j)} \quad , \qquad (1)$$

where $(ds^{(i)})^2 = dx_\mu^{(i)} dx^{(i)\mu}$, $R^2 = R_\mu R^\mu$, and $R_{ij} = x^{(i)} - x^{(j)}$. Notice that there is indeed no self-interaction and that mutual interactions occur only at light-like separations, yet time-symmetrically [9].

The next issue was to quantize the theory. Wheeler told Feynman not to pursue it, because he himself knew how to, whereas Pauli, who was skeptical about the whole thing, correctly predicted to him that Wheeler would not succeed. Feynman did not care. He saw the problem as a special case of a much more general one, namely that of quantizing a classical theory directly from its Lagrangian (or its action), i.e. without going through the Hamiltonian, and thus avoiding the approaches of both Schrödinger and Heisenberg. Somewhat fortuitously, Feynman learnt that Dirac had published an idea in this direction and actually even pointed out its worth in obtaining a manifestly relativistically covariant quantization. Feynman turned the idea into the path integral formulation of quantum mechanics. Still somewhat symbolically, it expresses the amplitude for a transition linking configurations q and q' of a systems between times t and t' as

$$\langle q', t' \mid q, t \rangle \propto \int_{\text{paths } \gamma} e^{iS(\gamma)} \mathcal{D}\gamma \, ,$$

where the integral ranges over all possible paths γ joining (q,t) with (q',t'). (Essentially, Dirac proposed the integrand, but not the integral.) That got Feynman a PhD.

Still in Princeton he then started work with Wilson on the separation of uranium isotopes but soon afterwards he accepted to join the Manhattan project in Los Alamos early in 1943 and under Bethe, for reasons that we shall see. There he attended to all sorts of tasks, such as managerial, theoretical and computational ones, including operating and fixing electromechanical computers, as well as overseeing human ones. His work on QED became intermittent.

In 1945 Feynman went to Cornell as a professor and in 1949 he finally accomplished his goal to quantize QED. He did so in two twin papers by the titles of *The Theory of Positrons* [5] and *Space-Time Approach to Quantum Electrodynamics* [6]. The first one actually did not deal with electrodynamics proper, but with the scattering of a Dirac particle, such as an electron, in an external electromagnetic field $A^\mu(x)$ depending on space and time, $x = (x^\nu)_{\nu=0,\dots 3}$. For comparison, scattering of a Schrödinger particle is computed perturbatively by means of the retarded Green's function of the free particle. That is not the only possible choice of Green's function, but a physically convenient one, because the state is usually specified in terms of an incoming state. Feynman noticed that that could not work for a Dirac particle, because the negative energy states are not available to the electron. Those states, which are to be interpreted as positive energy states of the positron, are available not after scattering, which would violate charge conservation, but before it, which describes pair annihilation. The correct Green's function would thus have to be like the retarded one in its positive energy part and like the advanced one in the the negative energy part; more pictorially a positron is an electron running backwards in time. That Green's function became later known as the Feynman propagator for the Dirac field and yields the weight associated to any of the straight electron/positron lines in the figure.

In the second paper Feynman quantized the theory defined by (1). The second term would be viewed as a perturbation of the first one and, in the resulting perturbation expansion, $\delta(R^2)$ would be the Green's function mediating the interaction between spacetime points x and y across their separation $R = x - y$. Feynman realized however that $\delta(s)$ would have to be replaced according to

$$\delta(s) \rightsquigarrow \delta_+(s) := \delta(s) + \frac{1}{i\pi s} = (i\pi)^{-1}(s - i0)^{-1} \, , \qquad (2)$$

which is the positive frequency part of $\delta(s)$. The result is essentially the Feynman propagator of the electromagnetic field, yielding the weight of the wiggly photon lines. That change retains the time symmetry of $\delta(R^2)$ (unlike the retarded or advanced Green's functions of which $\delta(R^2)$ is the half-sum), but now its future part (i.e. for $R^0 > 0$) has a time dependence like $\propto \delta_+(R^0 - |\vec{R}|)$ which contains only positive frequencies. That paralleled what he did in the previous paper and embodies the physical fact that the photons mediating the interaction only have positive energies.

The paper contained some guesswork. We may imagine that this could have been the cause for the poor reception, at least in Feynman's own perception, of his ideas at the Pocono conference the year before. He later explained the mishap by saying that "my machines came from too far away".

There was one more change in the paper with respect to. his work with Wheeler. The self-interaction $i = j$ was reinstated in (1) but was now less troublesome, because the δ-singularity got replaced by the milder pole singularity of δ_+. Both changes were acknowledged in two small footnotes, but were described in a much more flowery way years later in his Nobel lecture *The development of the space-time view in quantum electrodynamics*, which ended as follows: *So what happened to the old theory that I fell in love with as a youth? Well, I would say it's become an old lady, that has very little attractive left in her and the young today will not have their hearts pound anymore when they look at her. But, we can say the best we can for any old woman, that she has been a very good mother and she has given birth to some very good children. And, I thank the Swedish Academy of Sciences for complimenting one of them. Thank you.*

As a little aside, these words may prompt some considerations on Feynman's relation to women. It is likely that it would nowadays be frowned upon to use metaphors like the one just mentioned, at least if done by a man of his standing. A likely no-go today would also have been his frequent visits to those places, where anybody can meet lightly dressed women but only he could do physics ("When my calculations didn't work out, I would watch the girls.") One should however not jump to the conclusion that he reduced women to pleasure or motherhood. For instance, he married his first wife Arline Greenbaum at a time it was clear she would provide neither, as she was ill with tuberculosis and he knew she would soon die. He quit his job with Wilson at Princeton and joined the Manhattan project because he thought that the climate in New Mexico would be beneficial to her. (Sure, he also supported the goal of the project, which was then directed against Germany, but his last job in Princeton had the same one.) Arline taught him how to love. In a letter he would have liked to send her a year after her death in 1945, and which was opened after his own in 1988, he wrote "You, dead, are so much better than anyone else alive." On another count he encouraged his younger sister Joan to earn a PhD in physics, which she did. All this considered one should rather conclude that Feynman simply managed to bear life with a lightness perhaps forgotten nowadays.

But let us return to physics, at least briefly. Among the countless contributions across all of physics let us recall a few [1]: (i) On liquid helium, where he went beyond the phenomenological theory of Landau by placing its conclusions on the foundation provided by the many-body Schrödinger equation; (ii) on parity violation, where he and Gell-Mann proposed a modification of the Fermi theory of the weak interaction that would account for its then recent discovery. (The same V-A–theory was independently found by Sudarshan and Marshak); (iii) the parton hypothesis as a general model of composite hadrons. The theory proved successful in describing deep inelastic scattering of leptons by hadrons and is a precursor of the quark model and hence of Quantum Chromodynamics. (iv) Feynman, in a talk by the title *There's Plenty of Room at the Bottom* given in 1959, envisaged the opportunities that miniaturization would provide. He foresaw a lot of what nanotechnology would later accomplish, and ahead of Moore's law. (v) In a talk in 1981 he proposed to simulate quantum systems by universal computers that operate themselves according to the laws of quantum mechanics. While he may not be credited for being the first one to propose a quantum computer, he popularized the idea a lot. Many experiments, in particular with ultracold atoms, are done today in this spirit. However, those simulators are not universal and it remains to be seen whether other implementations are, while still being scalable and quantum.

The human side of Feynman is also well-known, not least thanks to himself, who spread lots of stories now collected in various books [2, 3, 4]. There is no need to recount them here (with one exception below). I just would like to add two anecdotes, not found there, told to me by Christoph Schmid and Klaus Hepp, who were young postdocs when they met him. Feynman took interest in the research of young fellows at Caltech, like Christoph, even if it was not in the line of his own. During discussions with them he expressed his views: To him physics had to have a clear "why", had to be reasonably concrete, and be done by getting one's hands dirty; moreover he liked to play the tough guy. As for Klaus, Feynman barely knew him, but one evening at a conference in Berkeley he entrusted him with safekeeping his office folder full of work in progress, so he could go to a topless bar. In the meantime Klaus had to resist all night the much bigger temptation of opening it.

Let us also spend some words on Feynman's relation to anything "cultured" or "intellectual", which in his youth included poetry ("To me no *real* man ever paid any attention to poetry and such things") but also higher mathematics, as we saw. At least in later years that was no longer always the case. One quite insightful instance, pointed out to me by Arthur Jaffe who witnessed it, is provided by a Rochester meeting in 1967, where Feynman proved to fully appreciate a genuine mathematical issue. Wightman lectured on quantum field theory in mathematical terms. After the lecture, Feynman asked a long question worth to be cited at least in part: ... *I want to get experimental results and I'm not usually worried about general mathematical questions. However there is a mathematical question which I think is very relevant. ... The question is whether this theory [QED] if carried out to the ultimate in all orders will give a satisfactory series (I don't mean in agreement with experiments, but with logic). ... I do not know and I am not at all convinced that ... the limit really does give a logically satisfactory theory; ... The question is, do we have a theory? ... I just don't know whether the whole thing means anything.* Some of the people present, including Hepp, could reassure him that the theory is finite order by order after renormalization, but that did not satisfy him: *I thought that each order could be computed and that the renormalization would work out. I didn't prove it and I am not worried about proving it. I thought it was true. I don't worry about proving something I think is true. I worry about proving something I'm not sure about.* Nobody could answer his real question, though. Reasons to worry (Landau pole, Gell-Mann-Low equation) existed already back then, but they had their basis in perturbation theory itself. Today, by works of Aizenman and Fröhlich, we know that the worries are well-founded on non-perturbative grounds.

In the last twenty-five years deep connections have emerged between Feynman diagrams on the one hand and Hopf algebras, Galois theory, and the Riemann zeta function on the other. One may just wonder how Feynman would have commented those outgrowths of his work into pure mathematics. For a change one might also wonder what poets would have thought about Feynman. It seems to me that Walt Whitman (1819-1892) would have seen in him the fulfillment of his own American dream (*Song of myself*): "Do I contradict myself? Very well then I contradict myself, (I am large, I contain multitudes.)"

We cannot close this note without making some links to Switzerland and specifically to Geneva. The first one simply is that he first met Gweneth Howarth, who would become his wife for the rest of his life, in 1958 in Geneva, where she spent time as an au-pair from the UK and he participated at a UN conference. The second link is a little Feynman story: During that same visit he lodged at a hotel which turned out to be a brothel. He made a reservation well in advance, perhaps still unaware of the kind of location, but was nonetheless delighted by what he then saw. That included the owner's embarassment as he became aware of the kind of guest once the UN called in. The reassuring thing however was "It's Switzerland; it was clean!". The last link is more important and relates to Ernst Stueckelberg, who was professor of theoretical physics in Geneva from 1935 to 1975. Fact is that already in 1934 Stueckelberg realized the importance of a manifestly relativistically covariant perturbation theory, and so is that in 1941 he drew figures in which positrons are represented by worldlines running backwards in time. The latter is acknowledged by Feynman in the abstract of his first paper of 1949. Stueckelberg and his student Rivier even came up with the Feynman propagator, but published it about one month after him. Less documented is the following event, reported by Mehra [7], which purportedly took place in 1965 after a lecture by Feynman: "After the lecture, Stueckelberg was making his way out alone ... from the CERN ampitheatre, when Feynman – surrounded by admirers – made the remark: 'He [Stueckelberg] did the work and walks alone toward the sunset; and, here I [Feynman] am, covered in all the glory, which rightfully should be his!'" Even if true, that would in no way diminish Feynman's scientific achievements. It would add to the man.

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